

Write your **name** in the box above.

Calculus Practice SAC

Question and Answer Book

VCE Methods Units 1&2

- Recommended writing time is **1 hour**
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Section A – Technology-free

Instructions

- Answer **all** questions in the spaces provided.
 - **No** technology (calculators or software) may be used.
 - In questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
 - In questions where more than one mark is available, appropriate working **must** be shown.
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Question 1 (9 marks)

A commuter train runs on a straight track. Station **B** is **450** m along the track from station **A**. The train leaves **A** from rest at time $t = 0$ seconds and comes to rest at **B** exactly **30** seconds later.

For $0 \leq t \leq 30$, the distance of the front of the train from **A** is modelled by $x(t) = pt^2 + qt^3$ metres, where p and q are constants.

- a. Show that $p = \frac{3}{2}$ and $q = -\frac{1}{30}$. 3 marks

- b. Find the maximum speed of the train between **A** and **B**, in km/h. 2 marks

- c. Find the average speed of the train between **A** and **B**. 1 mark

Question 1 continues on the next page.

- d.** Find the acceleration of the train **20** seconds after it leaves **A**. 1 mark

- e.** Find the times at which the speed of the train is **20** m/s. 2 marks

Section B – Technology-active

Instructions

- Answer **all** questions in the spaces provided.
 - One approved **technology** (calculator or software) may be used.
 - In questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
 - In questions where more than one mark is available, appropriate working **must** be shown.
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Question 2 (8 marks)

A level crossing is **126** m along the track from **A**, between **A** and **B**.

- a. Show that the front of the train reaches the level crossing at a time t that satisfies $t^3 - 45t^2 + 3780 = 0$.

1 mark

- b. Use Newton's method with $t_0 = 12$ to find t_1 .

2 marks

- c. Find, correct to two decimal places, the time at which the front of the train reaches the level crossing.

1 mark

- d. The train is **25** m long. The boom gates at the crossing come down **7** seconds before the front of the train reaches the crossing, and go up **3** seconds after the back of the train has passed it. Find the length of time for which the boom gates are down, correct to the nearest second.

3 marks

- e. Find the speed of the train as the back of the train passes the level crossing, in km/h correct to one decimal place.

1 mark

Question 3 (8 marks)

To shorten the trip, the operator considers a journey time of T seconds from A to B . The train still leaves A from rest and comes to rest at B , and its distance from A is still modelled by $x(t) = pt^2 + qt^3$, $0 \leq t \leq T$.

- a. Find p and q in terms of T .

2 marks

- b.** Show that the greatest magnitude of the acceleration of the train during the trip is $\frac{2700}{T^2} \text{ m/s}^2$.

2 marks

- c.** For passenger comfort, the magnitude of the acceleration must never be more than 3.6 m/s^2 . Find the least whole number of seconds that the trip can take.

2 marks

- d.** For the trip in part c, find the maximum speed of the train, in km/h correct to the nearest whole number.

2 marks

Question 4 (8 marks)

On a faster section of track further along the line, the train approaches station **C** at **25** m/s. The driver applies the brakes, and t seconds later the acceleration of the train is $-(0.3 + 0.02t)$ m/s², until the train stops.

- a.** Find the velocity of the train t seconds after the brakes are applied. 2 marks

- b.** Find the time taken for the train to stop, in seconds correct to two decimal places. 2 marks

- c.** The front of the train must stop at the end of the platform. Find how far from the end of the platform the driver must apply the brakes, correct to the nearest metre. 2 marks

- d. Find the time after the brakes are applied at which the train has covered half of its stopping distance. Give your answer in seconds correct to one decimal place. 2 marks

Question 5 (7 marks)

A straight road runs beside the track from **A** to **B**. A car travelling in the same direction as the train at a constant speed of u m/s passes station **A** at the instant the train leaves **A**. The train's distance from **A** is

again $x(t) = \frac{3t^2}{2} - \frac{t^3}{30}$ metres, $0 \leq t \leq 30$.

- a. Let $u = 16$. Find the exact times, for $0 < t \leq 30$, at which the front of the train is level with the car. 2 marks

- b. Let $u = 16$. Find the greatest distance by which the front of the train is ahead of the car, correct to the nearest metre. 2 marks

- c. Find the values of u for which the front of the train is level with the car exactly once for $0 < t \leq 30$.

3 marks
